

Statistical precursor signals for Dansgaard-Oeschger cooling transitions

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Abstract.

Given the likely bistability of the Atlantic Meridional Overturning Circulation (AMOC) and its recently inferred weakening, it is important to investigate the capability of identifying robust precursor signals for a possible future AMOC collapse as well as its collapses in the past. Dansgaard-Oeschger (DO) events, manifested most clearly as abrupt Northern-Atlantic temperature jumps during glacial conditions, likely reflect past switches between strong and weak AMOC modes. In general, the variance and the autocorrelation with a small lag increase in a system approaching a co-dimension one bifurcation point. Some previous studies find such statistical precursor signals for the DO warming transitions associated with a strengthening AMOC. On the other hand, statistical precursor signals for the abrupt DO cooling transitions, presumably associated with AMOC transitions from the strong to the weak mode, have not been identified. Here we identify robust and statistically significant precursor signals for several DO cooling transitions in Greenland ice core records. The important source of the statistical precursor signals stems from so-called rebound event, humps in the temperature observed at the end of interstadial, some decades to centuries prior to the transition. We propose several dynamical mechanisms that give rise to such rebound events and statistical precursor signals.

1 Introduction

A tipping point is a threshold in a forcing or control parameter, at which a small additional perturbation causes a qualitative change in the state of the system under consideration (Armstrong McKay et al., 2022). Once a tipping point is passed, a system can abruptly transition to an alternative stable or oscillatory state (Boers et al., 2022). The existence of climate tipping points and the possibility of abrupt transitions had been theoretically predicted since the early 1960s (Stommel, 1961; Boers et al., 2022). Paleoclimate evidence supports that abrupt climate changes due to crossing tipping points actually occurred in the past (Dakos et al., 2008; Brovkin et al., 2021; Boers et al., 2022). Recent observational and modelling evidence suggest the existence of climate tipping elements in the present Earth, which pose arguably one of the greatest risks in the context of anthropogenic global warming (Armstrong McKay et al., 2022; Boers et al., 2022). The Atlantic Meridional Overturning Circulation (AMOC) is considered one of the most important global-scale tipping elements (Armstrong McKay et al., 2022;

25 Brovkin et al., 2021; Boers et al., 2022). Recent studies have inferred that the AMOC is currently at its weakest in at least a millennium (Rahmstorf et al., 2015; Caesar et al., 2018) (see also Kilbourne et al. (2022) for possible uncertainties). Also the declining AMOC trend is projected to continue in the coming century, although the projections of the AMOC strength in the next hundred years are model-dependent (Masson-Delmotte et al., 2021).

Tipping point behaviors are mathematically classified into several types: bifurcation-induced, noise-induced, as well as rate-induced (Ashwin et al., 2012). The theory of critical slowing down (CSD) provides a framework to anticipate bifurcation-
30 induced transitions (Carpenter and Brock, 2006; Scheffer et al., 2009; Kuehn, 2013; Boers, 2018, 2021; Boers and Rypdal, 2021; Boers et al., 2022; Bury et al., 2020). The framework is based on the fact that the stability of a stable state is gradually lost as the system approaches the bifurcation point. Theoretically, the variance of the fluctuations around the fixed point diverges and the autocorrelation with a sufficiently small lag increases toward 1 at the critical point of a codimension-one bifurcation (Boers et al., 2022; Scheffer et al., 2009; Bury et al., 2020). Numerical studies show that these statistical precursor signals
35 (SPS) can anticipate simulated AMOC collapses (Boulton et al., 2014; Held and Kleinen, 2004). Moreover, based on further refined CSD indicators, evidence of stability loss in the North Atlantic in the course of the last century have been inferred from sea-surface temperature and salinity data, which might indicate that the inferred AMOC weakening is indeed associated with a loss of stability (Boers, 2021). To put these signals into context we investigate here if similar SPS can be detected also for past instances of AMOC collapse. We note that network-based SPS exploiting higher-dimensional dynamical information have also
40 been developed for predicting the AMOC collapse (Van Der Mheen et al., 2013; Feng et al., 2014).

Dansgaard-Oeschger (DO) events are millennial-scale abrupt climate transitions during glacial intervals (Dansgaard et al., 1993). They are most clearly imprinted in the $\delta^{18}\text{O}$ and calcium ion concentration $[\text{Ca}^{2+}]$ records from the Greenland ice cores (Fig. 1) (Rasmussen et al., 2014; Seierstad et al., 2014). The $\delta^{18}\text{O}$ and $[\text{Ca}^{2+}]$ are interpreted as proxies for site temperature and atmospheric circulation changes, respectively. DO warmings occur typically within a few decades and are followed by gradual
45 cooling during relatively warm glacial states termed interstadials, before a rapid return back to cold states referred to as stadials. The amplitude of the abrupt warming transitions ranges from 5 to 16.5°C (cf. Kindler et al. (2014) and references therein). While the detailed mechanism of DO events remains debated (Broecker et al., 1985; Ganopolski and Rahmstorf, 2001; Petersen et al., 2013; Dokken et al., 2013; Boers et al., 2018; Dima et al., 2018), recent studies with general circulation models suggest that DO oscillations can spontaneously arise from complex interactions between the AMOC, ocean stratification, atmosphere
50 and sea ice (Peltier and Vettoretti, 2014; Vettoretti et al., 2022; Brown and Galbraith, 2016; Klockmann et al., 2020; Zhang et al., 2021; Kuniyoshi et al., 2022; Malmierca-Vallet and Sime, 2023).

The DO events are considered the archetype of climate tipping behavior (Boers et al., 2022; Brovkin et al., 2021). Early works found an SPS based on autocorrelation for one specific DO warming, the onset of Bølling–Allerød interstadial (Dakos et al., 2008). In following works, the existence of SPS for DO warmings was questioned considering that DO warmings are
55 noise-induced rather than bifurcation-induced (Ditlevsen and Johnsen, 2010; Lenton et al., 2012). However a couple of later studies detected SPS for several DO warmings either by ensemble averaging of CSD indicators for many events (Cimatoribus et al., 2013) or by using Wavelet-transform techniques focusing on a specific frequency band (Rypdal, 2016; Boers, 2018). On the other hand, it has so far not been shown whether DO coolings are preceded by characteristic CSD-based precursor signals

as well. Given the recently inferred AMOC weakening (Rahmstorf et al., 2015; Caesar et al., 2018) and the observation-
60 based suggestion that this weakening may be related to stability loss (Boers, 2021), it is important to investigate if CSD-based
precursor signals can be detected for the DO cooling transitions as well, likely associated with past AMOC collapses. Though,
of course, predictability of past events does not necessarily imply any predictability in the future, especially given that the
recent AMOC weakening is likely driven by global warming and is thus from a mechanistic point of view different from past
AMOC weakenings in the glacial period.

65 In this study we report SPS for DO cooling transitions recorded in $\delta^{18}\text{O}$ and $\log_{10}[\text{Ca}^{2+}]$ (Seierstad et al., 2014; Rasmussen
et al., 2014) from three Greenland ice cores: NGRIP, GRIP and GISP2 (see Fig. 1 for NGRIP). The important source of
observed SPS stems from so-called rebound event, humps in the temperature proxy occurring at the end of interstadials, some
decades to centuries prior to the transition (Capron et al., 2010). When CSD indicators such as variance or lag-1 autocorrelation
are used for anticipating a tipping point, we conventionally assume that a system gradually approaches a bifurcation point.
70 However, if DO cycles are spontaneous oscillations as suggested in some GCMs (see above), in a strict sense there might
not be any bifurcation occurring around the timings of the abrupt transition in the DO cycles. Nevertheless, with conceptual
models of DO cycles, we demonstrate that CSD indicators may show precursor signals for abrupt transitions due to particular
dynamics near a critical point or a critical manifold.

The reminder of this paper is organized as follows. In Section 2, the data and method are described. In Section 3, we identify
75 robust and statistically significant SPS for several DO cooling transitions following interstadials longer than 1000 yr, and show
that rebound events prior to cooling transition are the source of observed statistical precursor signals. In Section 4 we discuss
the results by using conceptual models. A summary is given in Section 5.

2 Data and methods

2.1 Greenland ice core records

80 We explore CSD-based precursor signals for DO cooling transitions recorded in $\delta^{18}\text{O}$ and $\log_{10}[\text{Ca}^{2+}]$ (Seierstad et al., 2014;
Rasmussen et al., 2014) from three Greenland Ice Cores: NGRIP, GRIP and GISP2 (see Fig. 1 for NGRIP). Multiple records
are used for a robust assessment because each has regional fluctuations as well as proxy- and ice-core-dependent uncertainties.
The six records have been synchronized and are given at 20-yr resolution (Seierstad et al., 2014; Rasmussen et al., 2014).
They continuously span the last 104 kyr b2k (kiloyears before 2000 CE), beyond which only NGRIP $\delta^{18}\text{O}$ is available up to
85 a part of the Eemian interglacial. In addition, we use a version of the NGRIP $\delta^{18}\text{O}$ and dust records at 5-cm depth resolution
(epica community members, 2004; Gkinis et al., 2014; Ruth et al., 2003) in order to check the dependence of results on
temporal resolutions, with the caveat that these high-resolution records span only the last 60 kyr.

We follow the classification of interstadials and stadials and associated timings of DO warming and cooling transitions by
Rasmussen et al. (2014), where Greenland interstadials (stadials) are labelled with ‘GI’ (‘GS’) with few exceptions below. A
90 *rebound event* is an abrupt warming often observed before an interstadial abruptly ends (Capron et al., 2010) (arrows in Figs. 1,
2 and 3). Generally a long rebound event accompanies a long interstadial ($R^2 = 0.95$, Capron et al. (2010)). In Capron et al.

(2010) and Rasmussen et al. (2014), GI-14 and subsequent GI-13 are seen as one long interstadial with GI-13 considered to be a strongly expressed rebound event ending GI-14 because the changes in $\delta^{18}\text{O}$ and $\log_{10}[\text{Ca}^{2+}]$ during the quasi-stadial GS-14 do not reach the base-line levels of adjacent stadials. Similarly GI-23.1 and GI-22 are also seen as one long interstadial, and
95 GI-22 is regarded as a rebound event (and GS-23.1 as quasi-stadial) (Capron et al., 2010; Rasmussen et al., 2014). GI-20a is also recognized as a rebound event in Rasmussen et al. (2014). Given that the rebound events are warmings following a colder spell during interstadial conditions that does not reach the stadial levels (Rasmussen et al., 2014), we regard the following nine epochs as rebound-type events: GI-8a, the hump at the end of GI-11 (42240–~42500 yr b2k), GI-12a, GI-13, the hump at the end of GI-16.1 (56500–~56900 yr b2k), GI-20a, GI-21.1c-b-a (two warming transitions), GI-22 and GI-25a. When we
100 examine the effect of rebound events on our results, we exclude the entire parts including the cold spells prior to the rebound events.

The start (warming) and end (cooling) of each DO event are identified in 20-yr resolution based on both $\delta^{18}\text{O}$ and $[\text{Ca}^{2+}]$ in Rasmussen et al. (2014). The estimated uncertainty of event timing varies from event to event. We remove the 2σ uncertainty range of the event timing (40 to 400 years) estimated in Rasmussen et al. (2014) from our calculation of CSD indicators. It
105 effectively excludes parts of the transitions themselves from the calculation of the CSD indicators. Since the calculation of CSD indicators requires a minimum length of data points, we focus on interstadials longer than 1000 yr after removing 2σ uncertainty ranges of the transition timings, using 20-yr resolution data. This results in 12 DO interstadials to be investigated (Fig. 1, gray shades).

2.2 Statistical indicators of critical slowing down

110 Based on the theory of critical slowing down (CSD), we posit that the stability of a dynamical system perturbed by noise is gradually lost as the system approaches a bifurcation point (Boers et al., 2022; Scheffer et al., 2009; Bury et al., 2020). For the fold bifurcation, the variance of the fluctuations around a local stable state diverges and the autocorrelation function of the fluctuations increases toward 1 at any lag τ . The same is true for the transcritical as well as the pitchfork bifurcation (Bury et al., 2020). For the Hopf bifurcation, the variance increases, but the autocorrelation function of the form $C(\tau) = e^{|\mu|\tau} \cos \omega\tau$ may
115 increase or decrease depending on τ , where μ and $\pm\omega i$ are the real and imaginary parts of the complex eigenvalues of the Jacobian matrix of the local linearized system (Bury et al., 2020). Nevertheless the autocorrelation function $C(\tau)$ increases for sufficiently small τ . These characteristics can be used to anticipate abrupt transitions caused by codimension-one bifurcations. For time discrete series, we follow previous studies and calculate lag-1 autocorrelation.

Prior to calculating CSD indicators, we estimate the local stable state by using a local regression method called the locally
120 weighted scatterplot smoothing (LOESS) (Cleveland et al., 1992; Dakos et al., 2012). The polynomial degree is set to 1, i.e., the smoothing is performed with the local linear fit. The smoothing span parameter α that defines the fraction of data points involved in the local regression is set to 50% of each interstadial length in a demonstration case, but we examine the dependence of results on α over the range 20–70%. The difference between the record and the smoothed one gives the residual fluctuations. The CSD indicators, i.e., variance and lag-1 autocorrelation, are calculated for the residuals over a rolling window. The size

125 W of this rolling window is set to 50% of the length of the considered interstadial part of the records in each case. To test robustness this is changed over the range 20–60%.

The statistical significance of precursor signals of critical transitions, in terms of positive trends of CSD indicators, is assessed by hypothesis testing (Theiler et al., 1992; Dakos et al., 2012; Rypdal, 2016; Boers, 2018). We consider as null model a stationary stochastic process with preserved variance and autocorrelation. The n surrogate data are prepared from the original residual series by the phase-randomization method, thus preserving the variance and autocorrelation function of the original time series via the Wiener-Khinchin theorem. Here we take $n = 1000$. The linear trend (a_o) of the CSD indicator for the original time series and the linear trends (a_s) of CSD indicators for the surrogate data are calculated. We consider the precursor signal of the original time series statistically significant if the probability of $a_s > a_o$ (p -value) is less than 0.05.

3 Results

135 3.1 Characteristic precursor signals of DO coolings

As CSD indicators we consider the variance and lag-1 autocorrelation, calculated in rolling windows across each interstadial. The 12 interstadials longer than 1000 yr are magnified in Figs. 2 and 3 (top rows, blue) for the NGRIP $\delta^{18}\text{O}$ record. See Figs. S1–S10 for the other records. For each interstadial, the nonlinear trend is estimated with the locally weighted scatterplot smoothing (Cleveland et al., 1992; Dakos et al., 2012) (Figs. 2 and 3, top row, red). In this case the smoothing span α that defines the fraction of data points involved in the local regression is set to 50% of each interstadial length. Gaussian kernel smoothing gives similar results. The difference between the record and the nonlinear trend gives the approximately stationary residual fluctuations (second row). The CSD indicators are calculated from the residual series over a rolling window. In Figs. 2 and 3 the rolling window size W is set to 50% of each interstadial length (a default value in Dakos et al. (2008)). The smoothing span α and the rolling window size W are taken as fractions of individual interstadial length because time scales of local fluctuations (such as the duration of rebound events) change with the entire duration of interstadial. We examine the dependence of the results on α and W as part of our robustness tests.

The variance is plotted in the third row of Figs. 2 and 3. Positive trends in the variance are observed for 9 out of 12 interstadials; the individual trends are statistically significant in 6 out of 12 cases ($p < 0.05$), based on a null model assuming the same overall variance and autocorrelation, constructed by producing surrogates with randomized Fourier phases. The lag-1 autocorrelation is also plotted for the same data in the bottom row. Positive trends in lag-1 autocorrelation are observed for 10 out of 12 interstadials, but are statistically significant only in 2 cases ($p < 0.05$). Just a positive trend without significance cannot be considered a reliable SPS, but if one indicator has a significantly positive trend, the other indicator with consistently positive trend may at least support the conclusion (e.g., GI-19.2 and GI-14-14 in Fig. 3). In several cases (GI-24.2, 21.1, 16.1, 14-13 and 12), the lag-1 autocorrelation first decreases and then increases. The drastic increases in both indicators near the end of the interstadials reflect the rebound events (arrows in Figs. 2 and 3). We obtain similar results for the other ice core records (Figs. S1–S10). While we observe a number of positive trends for all the records, the number of detected statistically significant trends depends on the record and CSD indicator (Table S1).

We check robustness of our results against changing smoothing span α and rolling window size W (Dakos et al., 2012). We calculate the p -value for the trend of each indicator changing the smoothing span between 20–70% of interstadial length (in steps of 10%) and the rolling window size between 20–60% (also in 10% steps), respectively. This yields a 6×5 matrix for the p -values: Example results, for GI-25 and $\delta^{18}\text{O}$, are shown in Figs. 4a (variance) and 4b (lag-1 autocorrelation). The cross mark (x) indicates significant positive trends ($p < 0.05$) and the small open circle (o) indicates positive trend that are significant at 10% confidence but not at 5% confidence. Full results for the 12 interstadials, 6 records, and two CSD indicators are shown in Figs. S11–S22. We consider positive trends in CSD indicators, i.e. the SPS of the transition, to be overall robust if we obtain significant positive trends ($p < 0.05$) for at least half (≥ 15) of the 30 parameter sets.

The robustness analysis is performed for all the long interstadials of the 6 records and the two CSD indicators (Fig. 4d). Among the 12 interstadials, we find at least one robust SPS for 9 interstadials (GI-25, 23.1, 21.1, 20, 19.2, 16, 14, 12 and 8) and multiple robust SPS for 6 (GI-25, 23.1, 21.1, 14, 12 and 8). If the data series is a stationary stochastic process, the probability of observing a robust SPS is estimated to be 5% (Appendix A). Thus the observation of robust SPS for 36 cases (31%) over the 116 combinations of data and indicators in Fig. 4d is very unlikely to occur by chance. Robust SPS are more likely to be observed in longer interstadials than in shorter ones (compare Figs. 4c and 4d).

3.2 Further sensitivity analyses

We examine how much the rebound events affect the detection of CSD-based SPS. For this purpose CSD indicators are again calculated excluding the rebound events and their preceding cold spells (see Section 2.1). Only four interstadials (GI-23.1, 14, 12 and 8) exhibit robust SPS without the rebound events, while 8 interstadials (GI-25, 23.1, 21.1, 20, 16, 14, 12 and 8) exhibit robust SPS with the rebound events included (Fig. S23). The rebound events should hence be considered important, sometimes indispensable, sources for SPS of DO coolings.

We also examine the dependence of the results on the time resolution of the data. Here we use a high-resolution (5-cm depth) $\delta^{18}\text{O}$ record (epica community members, 2004; Gkinis et al., 2014) and a dust record (Ruth et al., 2003) from the NGRIP over the last 60 kyr. Since the data in these records are non-uniform in time, they are linearly resampled every 5 yr before calculating CSD indicators. We focus on 11 interstadials longer than 300 yr in order to have enough data points. For the dust record, three interstadials (GI-15, 8 and 7) are excluded from the analysis because the original data has long parts of missing values. The CSD indicators, calculated with a smoothing span of $\alpha = 50\%$ and rolling windows with $W = 50\%$, are shown in Figs. S24–S27. Through the robustness analyses with respect to α and W , we find at least one robust SPS for 3 out of 11 interstadials (Fig. S28). The robust SPS for GI-14-13 and GI-12 from the high-resolution records are consistent with those from the 20-yr resolution records. Moreover for GI-1, the high-resolution $\delta^{18}\text{O}$ record exhibits a robust SPS in terms of lag-1 autocorrelation, although the 20-yr resolution record does not. Again, shorter interstadials ($\lesssim 1000$ yr) do not show robust SPS.

4 Possible dynamical mechanisms

We detected robust precursor signals of DO cooling transitions for more than half of the long (>1000 yr) interstadials, but not for shorter interstadials. The results suggest that long interstadials, the existence of rebound events, and the presence of SPS for the DO cooling transitions are all related (except for GI-19.2, which has no noticeable rebound event). These aspects may be related to generic properties of nonlinear dynamical systems. On the basis of conceptual mathematical models, we discuss four possible dynamical mechanisms leading to the precursor signals of DO cooling transitions. In three of four mechanisms, oscillations like the rebound events can systematically arise prior to the abrupt cooling transitions. These modelling results justify the inclusion of the rebound events in the search for precursor signals presented above. Unless otherwise mentioned, details on model parameters as well as conducted hysteresis experiments are given in Appendix B.

1. The fold bifurcation mechanism. Since the pioneering work by Stommel (1961), the AMOC is considered to exhibit bistability (Rahmstorf, 2002). The bistability of the AMOC strength x may be conceptually modelled by a double-fold bifurcation model: $\dot{x} = f(x) + p + \xi(t)$, where $f(x)$ has two fold points like $x - x^3$ and $|x|(1 - x)$. Here we take the quadratic from $f(x) = |x|(1 - x)$, but the following arguments are qualitatively the same for $x - x^3$. The parameter p represents the surface salinity flux (i.e. negative freshwater flux), and $\xi(t)$ denotes white Gaussian noise. The unperturbed model for $\xi(t) = 0$ has equilibria on an S-shaped curve: $f(x) + p = 0$ (Fig. 5a, green). The state $x(t)$ initially on the upper stable branch jumps down to the lower stable branch as p decreases across the fold bifurcation point at $p = -0.25$. It is well known that the variance and the autocorrelation of the local fluctuations (i.e. CSD indicators) increase as p approaches the fold bifurcation point since the restoring rate toward the stable state decreases (Scheffer et al., 2009; Boers et al., 2022).

2. Stochastic slow-fast oscillation mechanism. The FitzHugh-Nagumo (FHN) system is a prototypical model for slow-fast oscillations and excitability (FitzHugh, 1961; Nagumo et al., 1962). It is often invoked for conceptual models of DO oscillations (Rial and Yang, 2007; Kwasniok, 2013; Roberts and Saha, 2017; Mitsui and Crucifix, 2017; Lohmann and Ditlevsen, 2019; Riechers et al., 2022; Vettoretti et al., 2022). An FHN-type model of DO oscillations is obtained by introducing a slow variable y into the fold bifurcation model: $\dot{x} = \frac{1}{\tau_x} (|x|(1-x) + y + p) + \xi(t)$, $\dot{y} = \frac{1}{\tau_y} (-x - y)$, where τ_x and τ_y are time-scale parameters ($\tau_x \ll \tau_y$). Invoking the salt-oscillator hypothesis for DO oscillations suggested by the comprehensive climate model simulations that are successful in reproducing DO cycles (Vettoretti and Peltier, 2018), we may interpret y as the salinity in the polar halocline surface mixed layer, which gradually decreases (increases) when the AMOC intensity x is strong (weak). Here we consider the case that the unperturbed system is excitable. For example for $p = 0.26$, the unperturbed system has a stable equilibrium near the upper fold point of the S-shaped critical manifold, $\{(x, y) \in \mathbb{R}^2 \mid y = -|x|(1-x) - p\}$ (Fig. 5c, green), but stochastic oscillations sustain under the dynamical noise forcing $\xi(t)$ (Figs. 5b and 5c, blue). Due to the time-scale separation ($\tau_x \ll \tau_y$), the oscillations occur along the attracting parts of the critical manifold (Fig. 5c). Because y is much slower than x , the dynamics of x is similar to the dynamics of the fold bifurcation model with slowly changing y . Consequently SPS can be effectively observed near the fold point of the critical manifold (Fig. S29). The increase of the variance prior to the transitions in the FHN model is reported also in Meisel and Kuehn (2012). Since the unperturbed system has an equilibrium near the upper fold point, the motion is stagnant near the fold point. This provides favorable conditions for observing SPS. The state jumps

from the upper branch of the critical manifold to its lower branch often after an upward jump induced by noise. These upward jumps resemble the rebound events prior to DO coolings. The overall phenomenon is the same in the self-sustained oscillation regime of the FHN model as long as the equilibrium locates near the fold point of the critical manifold.

225 3. Hopf bifurcation mechanism. In contrast to the fold bifurcation, the Hopf bifurcation manifests oscillatory instability (Strogatz, 2018). In several ocean box models, the thermohaline circulation is destabilized via a Hopf bifurcation (Alkhayoun et al., 2019; Lucarini and Stone, 2005; Abshagen and Timmermann, 2004; Sakai and Peltier, 1999). It is also considered a potential generating mechanism of DO oscillations in a low-order coupled climate model (Timmermann et al., 2003) and in a comprehensive climate model (Peltier and Vettoretti, 2014). Assume that the parameter p decreases slowly in the FHN-type
230 model (Figs. 5d and 5e). The underlying dynamics changes from the stable equilibrium to the limit-cycle oscillations at the Hopf bifurcation point $p = (1 - \tau_x/\tau_y)^2/4$ (Appendix B). If stochastic forcing is added to the system, noise-induced small oscillations can appear prior to the onset of the limit-cycle oscillations (Fig. 5d and 5e). The precursor oscillations resemble rebound events, while their shape depends on the noise. Again SPS can be observed near the Hopf bifurcation point (Fig. S30) (Bury et al., 2020; Meisel and Kuehn, 2012; Boers et al., 2022).

235 4. Mixed-mode oscillation mechanism. Mixed-mode oscillations (MMOs) are periodic oscillations consisting of small and large-amplitude oscillations (Koper, 1995; Desroches et al., 2012; Berglund and Landon, 2012). They often arise in systems with one fast variable and two slow variables (Desroches et al., 2012). In this regard, we can extend the above FHN-type model to exhibit MMOs, for example, as follows: $\dot{x} = \frac{1}{\tau_x}(|x|(1-x) + y + p)$, $\dot{y} = \frac{1}{\tau_y}(-x - y + k(z - y))$ and $\dot{z} = \frac{1}{\tau_z}(-x - z + k(y - z))$, where z is another slow variable with time scale $\tau_z (\gg \tau_x)$ and k is the diffusive-coupling constant between
240 slow variables. We interpret y as the surface salinity in the North Atlantic convection region that directly affects the AMOC strength x again, and z as the surface salinity outside the convection region that affects the surface salinity y in the convection region via mixing. This extended FHN-type model is introduced here just for demonstrating that MMOs may appear in a FHN-type model with a minimal dimensional extension. For certain parameter settings (Appendix B), the system has an unstable equilibrium $(x, y, z) = (\sqrt{p}, -\sqrt{p}, -\sqrt{p})$ of saddle-focus type, with one stable direction with a negative real eigenvalue and a
245 two-dimensional unstable manifold with complex eigenvalues with positive real part. The slow-fast oscillations occur along the critical manifold $\{(x, y, z) \in \mathbb{R}^3 \mid y = -|x|(1-x) - p\}$ (Figs. 5f and 5g). However, due to the saddle-focus equilibrium on the critical manifold, the trajectory is attracted toward the saddle from the direction of the stable manifold (black segment) and repelled from it in a spiralling fashion. The striking point is the systematic occurrence of small-amplitude oscillations prior to the abrupt transition, which also resemble the rebound events prior to the DO cooling transitions. A more realistic time series
250 is obtained if an observation noise is added on $x(t)$ (Fig. S31). Then SPS can be stably observed near the fold point of the critical manifold.

Based on the four types of simple mathematical models, we have proposed four possible dynamical mechanisms for the DO cooling transitions that can manifest statistical precursor signals (SPS) and behavior resembling the rebound events found in the ice core records: The SPS for the DO coolings can be (i) strict critical slowing down due to the approaching of a fold
255 bifurcation, (ii) critical slowing down in a wider sense, in stochastic slow-fast oscillations, (iii) noise-induced oscillations prior to Hopf bifurcations, or (iv) the signal of mixed-mode oscillations. While the details of these mechanisms are different, they

are all related to the fold points of the equilibrium curve or the critical manifolds. Consequently the precursor signals can be detected by the conventional CSD indicators. Note that the precursor behavior like the rebound events occur when the modelled interstadials have equilibria on the critical manifold with marginal stability (i.e., the equilibrium has neither strong stability leading to a permanent state nor strong instability leading to brief interstadials) (Figs. 5b–5g). Then the duration of the modelled interstadial is comparatively long in relation to the marginal stability. This provides an explanation why the rebound events and the SPS are observed only for long interstadials ($\gtrsim 1000$ yr) in the ice core records.

5 Summary

In this study we have explored statistical precursor signals (SPS) for Dansgaard-Oeschger (DO) cooling transitions following interstadials longer than 1000 years, using the six Greenland ice core records and the two critical slowing down (CSD) indicators (variance and lag-1 autocorrelation). Among the 12 interstadials, we find at least one robust SPS for 9 interstadials (GI-25, 23.1, 21.1, 20, 19.2, 16, 14, 12 and 8) and multiple robust SPS for 6 (GI-25, 23.1, 21.1, 14, 12 and 8) (Fig. 4d). Rebound events prior to the cooling transitions are important in producing the SPS. We have proposed four different dynamical mechanisms to explain the appearance of rebound events and the observed SPS: (i) strict critical slowing down due to the approaching of a fold bifurcation, (ii) critical slowing down in a wider sense, in stochastic slow-fast oscillations, (iii) noise-induced oscillations prior to Hopf bifurcations, or (iv) the signal of mixed-mode oscillations. In the latter three mechanisms, oscillations like the rebound events can systematically arise prior to the abrupt cooling transitions. A recent study using an ocean general circulation model shows that a rebound-event-type behavior of AMOC is caused by a so-called intermediate tipping behavior due to multiple stable ocean circulation states that exist near but prior to the tipping point, leading to a significant AMOC weakening (Lohmann et al., 2023). It thus suggests a high-dimensional origin of rebound events, which would be beyond the possible low-dimensional dynamical mechanisms proposed in this study. Because the DO cooling transitions are most likely associated with AMOC collapses, our results provide evidence from past climate records that it could indeed be possible to anticipate AMOC transitions from the strong to the weak mode based on CSD indicators.

Supplement

The supplement related to this article is available online at: https://github.com/takahito321/Predictability-of-DO-cooling/blob/main/EWS_DO_cooling_SI.pdf

Code and data availability. The Greenland ice core records used in this study can be obtained from <https://www.iceandclimate.nbi.ku.dk/data/>. The R-codes used in this study are available from the corresponding author upon request.

Appendix A: Probability of observing robust precursor signals

285 The statistical significance of precursor signals of critical transitions, in terms of positive trends of CSD indicators, is assessed by hypothesis testing (Theiler et al., 1992; Dakos et al., 2012; Rypdal, 2016; Boers, 2018). We consider as null model a stationary stochastic process with preserved variance and autocorrelation. The n surrogate data are prepared from the original residual data series by the phase-randomization method, thus preserving the variance and autocorrelation function of the original time series via Wiener-Khinchin theorem. Here we take $n = 1000$. The linear trend (a_0) of the CSD indicator for the original residual time series and the linear trends (a_s) of CSD indicators for the surrogate data are calculated. We consider the precursor signal of the original series statistically significant if the probability of $a_s > a_0$ (p -value) is less than 0.05. Thus, if the original data is already a stationary stochastic process (exhibiting no CSD), one should expect spuriously significant results at a probability of 0.05 by definition. In principle this is independent of the smoothing span α as well as the rolling window size W used for calculating CSD indicators. We consider a precursor signal robust if we find at least 15 significant cases ($p < 0.05$) for 30 combinations of α and W . Then the probability of observing a robust precursor signal can be shown to be 0.05. In order to check this numerically, we generate 5000 surrogates of the original $\delta^{18}\text{O}$ series of interstadial GI-25 and calculate the probability of finding robust precursor signals. The resulting fractions are 0.044 for the variance and 0.042 for the lag-1 autocorrelation, which are close to 0.05. For the case of GI-12, we obtain 0.042 for the variance and 0.052 for the lag-1 autocorrelation, again close to 0.05. These results support that the probability of observing a robust precursor signal is 5% if the data are stationary stochastic processes.

Appendix B: Details of conceptual models used in Fig. 5

Here we describe specific settings for four conceptual models representing different candidate mechanisms for the DO cooling transitions. Unless otherwise mentioned, stochastic differential equations below are solved with the Euler-Maruyama method with step size of 10^{-3} .

305 1. The bistability of the AMOC strength x can be conceptually modelled by a double-fold bifurcation model: $\dot{x} = f(x) + p + \xi(t)$, where $f(x)$ has two fold points, here for f one can use either $f(x) = x - x^3$ or $f(x) = |x|(1 - x)$. We take the quadratic function $f(x) = |x|(1 - x)$ that arises in Stommel (1961) model. p represents a forcing parameter on the AMOC strength x , e.g., salinity forcing on the North Atlantic (i.e., negative freshwater forcing). $\xi(t)$ is white Gaussian noise, e.g. freshwater perturbations or weather forcing. In Fig. 5a, the initial condition is taken at $x(0) = 1.1$, near the upper stable fixed point of the unperturbed system. The parameter p is then slowly decreased from 0.1 to -0.4 over the period from $t = 0$ to $t = 500$, to trigger the bifurcation induced transition.

315 2. The FitzHugh-Nagumo-type (FHN-type) system is a prototypical model of slow-fast oscillators (FitzHugh, 1961; Nagumo et al., 1962) and often invoked for conceptual models of DO oscillations (Rial and Yang, 2007; Kwasniok, 2013; Roberts and Saha, 2017; Mitsui and Crucifix, 2017; Lohmann and Ditlevsen, 2019; Riechers et al., 2022; Vettoretti et al., 2022). The FHN-type model subjected to dynamical noise is obtained by introducing a slow variable y into the fold bifurcation model: $\dot{x} = \frac{1}{\tau_x}(|x|(1 - x) + y + p) + \xi(t)$, $\dot{y} = \frac{1}{\tau_y}(-x - y)$, where τ_x and τ_y are time-scale parameters ($\tau_x \ll \tau_y$). Following the salt-

oscillator hypothesis of to explain DO cycles (Vettoretti and Peltier, 2018), we may interpret y as the salinity in the polar halocline surface mixed layer, which decreases (increases) when the AMOC is strong (weak). In the case of Fig. 5 we set $p = 0.26$, $\tau_x = 0.01$, $\tau_y = 1$ and $\langle \xi_x^2 \rangle = 0.3$. The x -nullcline (critical manifold) of the unperturbed system is $y = -|x|(1-x) - p$ (Fig. 5c, green) and the y -nullcline is the $y = -x$ (Fig. 5c, magenta dashed). The intersection of the x - and y -nullclines is the equilibrium point of the unperturbed system $(\sqrt{p}, -\sqrt{p})$, which is near the fold point of the critical manifold in this parameter setting.

3. For demonstrating the Hopf bifurcation mechanism in Figs. 5d and 5e, the same stochastic FHN-type model is used with $\tau_x = 0.01$, $\tau_y = 1$ and $\langle \xi^2 \rangle = 0.05$, but here p is gradually decreased from 0.3 to 0.2, over a period of 10 time units. For $0.2 < p < 0.3$, the system has a unique equilibrium point at $(x, y) = (\sqrt{p}, -\sqrt{p})$. The Hopf bifurcation of an equilibrium occurs if the complex eigenvalues of the Jacobian matrix at the equilibrium passes the imaginary axis (Strogatz, 2018). The eigenvalues of the Jacobian matrix at this equilibrium are $\lambda_{\pm} = \frac{1}{2} \left\{ \frac{1-2\sqrt{p}}{\tau_x} - \frac{1}{\tau_y} \pm \sqrt{\left(\frac{1-2\sqrt{p}}{\tau_x} - \frac{1}{\tau_y} \right)^2 - \frac{8\sqrt{p}}{\tau_x \tau_y}} \right\}$. These eigenvalues λ_{\pm} are complex conjugates for $\frac{1}{4} \left(1 + \frac{\tau_x}{\tau_y} - 2\sqrt{\frac{\tau_x}{\tau_y}} \right)^2 < p < \frac{1}{4} \left(1 + \frac{\tau_x}{\tau_y} + 2\sqrt{\frac{\tau_x}{\tau_y}} \right)^2$. In this range of p , the real part of λ_{\pm} changes from negative to positive at the Hopf bifurcation point: $p_H = \frac{1}{4} \left(1 - \frac{\tau_x}{\tau_y} \right)^2$. For $\tau_x/\tau_y = 0.01$, $p_H \approx 0.245$.

4. The mixed-mode oscillation model is obtained if the FHN-type model is extended to have multiple interacting slow variables. E.g., $\dot{x} = \frac{1}{\tau_x} (|x|(1-x) + y + p)$, $\dot{y} = \frac{1}{\tau_y} (-x - y + k(z - y))$ and $\dot{z} = \frac{1}{\tau_z} (-x - z + k(y - z))$, where z is another slow variable with time scale $\tau_z (\gg \tau_x)$ and k is the diffusive coupling constant between slow variables. We interpret y as the surface salinity in the North Atlantic convection region, which directly affects the AMOC strength x , and z as the surface salinity outside the convection region that affects the surface salinity y in the convection region via mixing. We set $\tau_x = 0.02$, $\tau_y = 2$, $\tau_z = 4$, $p = 0.225$ and $k = 0.8$. This system has an unstable equilibrium $(x, y, z) = (\sqrt{p}, -\sqrt{p}, -\sqrt{p})$ of saddle-focus type, with one stable direction with a negative real eigenvalue -0.67 and a two-dimensional unstable manifold with two complex conjugate eigenvalues with positive real part $0.94 \pm 4.7i$.

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340 *Competing interests.* The authors declare that they have no competing financial interests.

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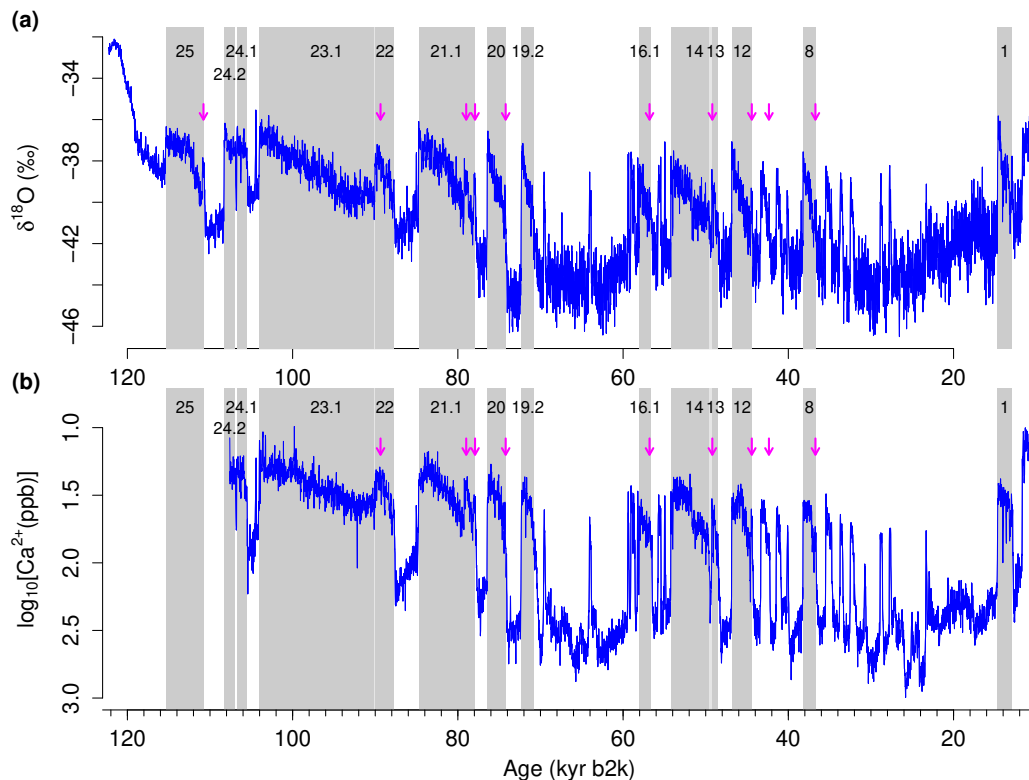


Figure 1. Greenland records from the NGRIP ice core: (a) $\delta^{18}\text{O}$ and (b) $\log_{10}[\text{Ca}^{2+}]$ (Seierstad et al., 2014; Rasmussen et al., 2014). The interstadial parts longer than 1000-yr are highlighted with grey shades; their numbering is given at the top of each record (Rasmussen et al., 2014). The rebound events are indicated by arrows (see Section 2.1 for their list). Both records are presented at 20-yr resolution. The $\log_{10}[\text{Ca}^{2+}]$ record is available only up to DO-24.1. The compositions of GI-23.1 and GI-22, as well as of GI-14 and GI-13, are considered individual long interstadials (Capron et al., 2010; Rasmussen et al., 2014). The vertical axis for $\log_{10}[\text{Ca}^{2+}]$ in (b) is reversed to ease visual comparison with the $\delta^{18}\text{O}$ record.

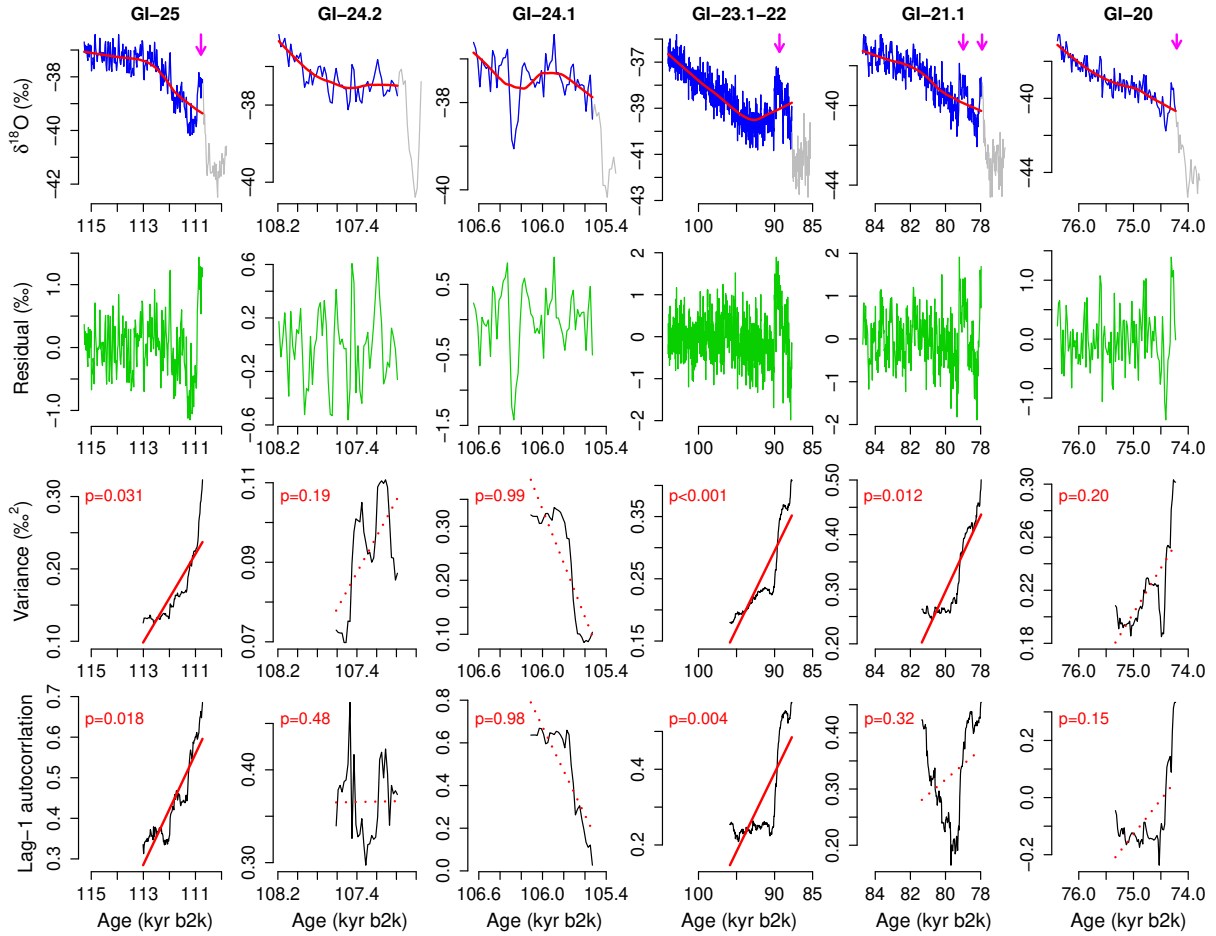


Figure 2. Analysis of CSD-based precursor signals of abrupt DO cooling transitions, for the first 6 interstadials of NGRIP $\delta^{18}\text{O}$, from 115 ka to 74 kyr b2k. (Top row) Interstadials longer than 1000 yr (blue). The cooling transition and stadial parts are shown in grey (Rasmussen et al., 2014). Nonlinear trends are calculated with the Locally Weighted Scatterplot Smoothing (LOESS) (red). The smoothing span α that defines the fraction of data points involved in the local regression is set to 50% of each interstadial length. The rebound events are indicated by arrows (see Section 2.1). (Second row) Residuals (green) resulting from subtracting the nonlinear trends (red) from the records (blue). (Third row) Variance estimate in rolling windows (black) with size equal to 50% of each interstadial length. Values are plotted at the right edge of each rolling window. The linear trend is shown by a solid red line for $p < 0.05$, by a dashed red line for $0.05 < p < 0.1$, and by a dotted line for $p > 0.1$. (Fourth row) Same as third row but for the lag-1 autocorrelation (i.e., a lag of 20 yr).

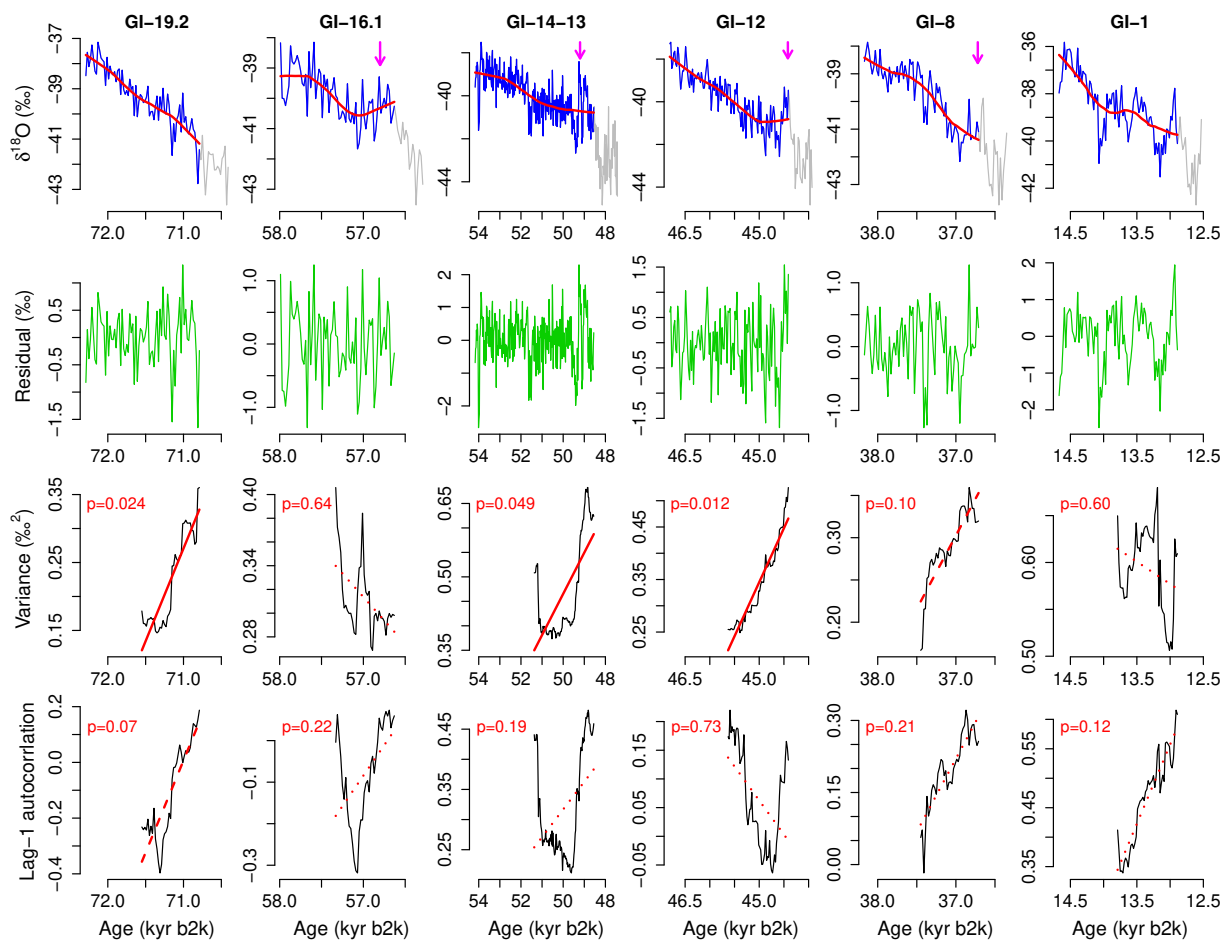


Figure 3. Same as Fig. 2 but for the following 6 interstadials, from 74 ka to 12 kyr b2k.

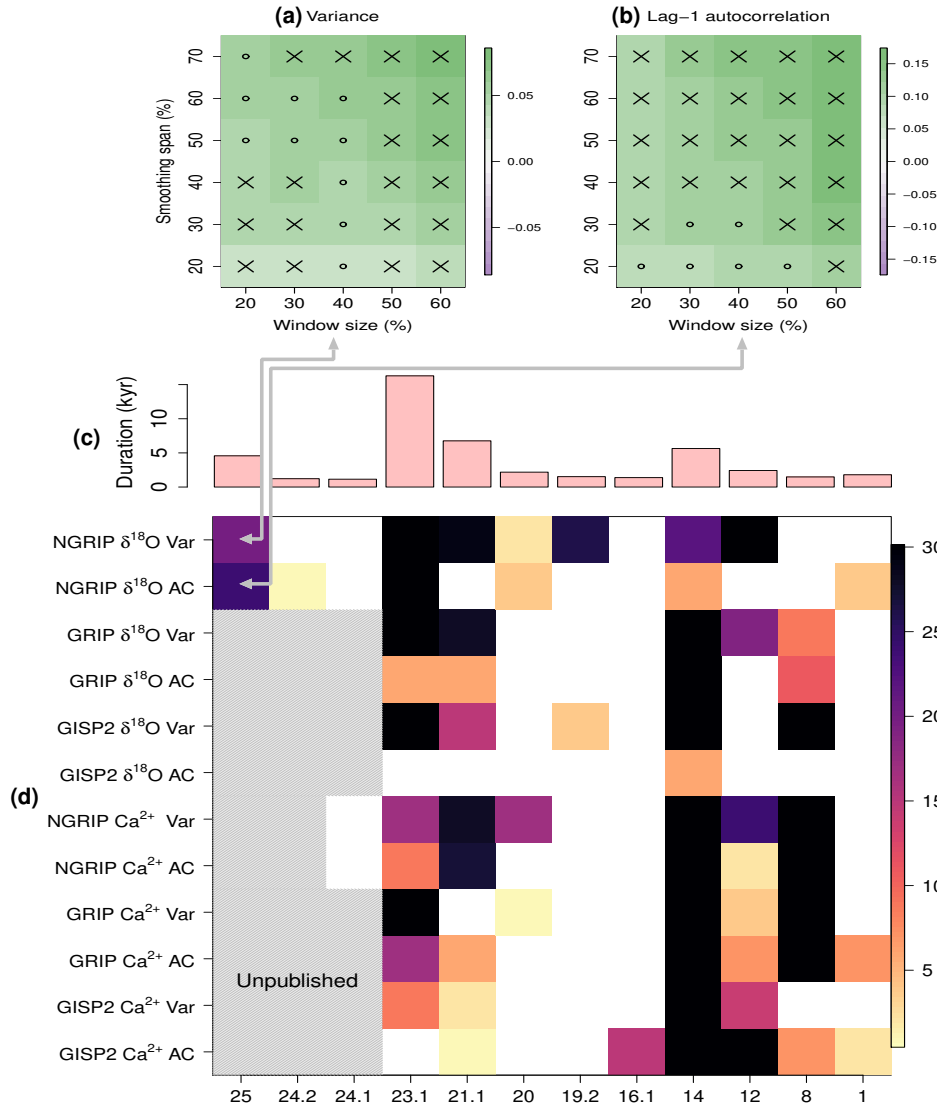


Figure 4. Detection of precursor signals of DO cooling transitions for different interglacials, different proxy variables, different ice cores, and different CSD indicators. (a,b) Robustness analysis of precursor signals with respect to the smoothing span and the rolling window size (% of interstadial length): the case of GI-25 interstadial from the NGRIP $\delta^{18}\text{O}$ record. The CSD indicator is the variance in (a) and the lag-1 autocorrelation in (b). Cross marks (x) indicates statistically significant positive trend of the respective CSD indicator ($p < 0.05$) based on a phase surrogate test (see Section 2.2), small open circles (o) indicate barely significant positive trends ($0.05 < p < 0.1$) and cells are left blank if $p > 0.1$. (c) Durations of interstadials longer than 1000 yr. (d) Robustness of finding precursor signals for DO cooling transitions. The color indicates the number significant ($p < 0.05$) positive trends in each of the 30 sets of the smoothing spans and the rolling window sizes as in (a) and (b). For the cases of grey-shaded cells, the data is not publicly available.

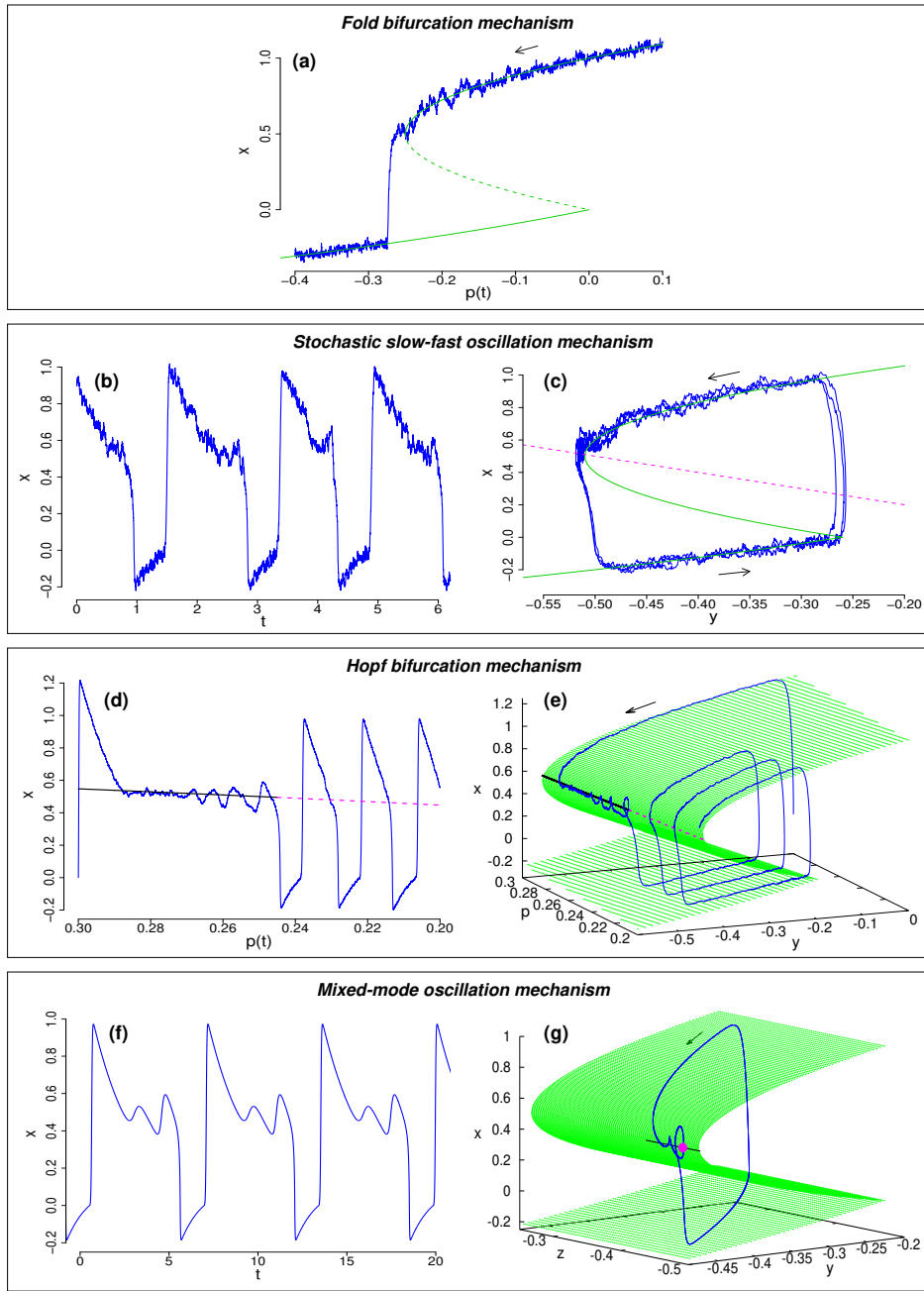


Figure 5. Four potential dynamical mechanisms for the DO cooling transitions: (a) Fold bifurcation mechanism. The time series $x(t)$ for decreasing $p(t)$ (blue). The green lines show the stable (solid) and unstable (dashed) fixed points. (b,c) Stochastic slow-fast oscillation mechanism of a FHN-type model. An example time series $x(t)$ is shown in (b) and the phase space trajectory (blue) in (c); the x -nullcline, i.e., the critical manifold, is shown in green. The y -nullcline is shown in dashed magenta. (d,e) Hopf bifurcation mechanism. An example time series $x(t)$ is shown in (d) as a function of $p(t)$. Stable (black, solid) and unstable (magenta, dashed) fixed points are also shown. The corresponding phase space trajectory $(x(t), y(t))$ for decreasing p is shown in (e) in blue. The critical manifold (green). (f,g) Mixed-mode oscillation mechanism. An example time series $x(t)$ is shown in (f) and the corresponding phase space trajectory in (g). The magenta dot is the saddle point with a stable manifold in the direction of the black segment; the trajectory is spiralling around it.